



Annual Progress Report for  
ONR Grant No. N00014-96-1-0930

Fractal Point Process and Queueing Theory  
and Application to Communication Networks

*for the period*

June 1, 1996 through May 31, 1997

Principal Investigator: Prof. Gregory W. Wornell

Research Laboratory of Electronics  
Massachusetts Institute of Technology  
Cambridge, MA 02139-4307

In the past year, our broad program of research has continued to explore efficient solutions fundamental problems of communication for wireless and wired networks, exploiting interrelated perspectives from communication theory, information theory, signal processing theory, and control theory. As part of this work novel applications chaos and fractal geometry are also being explored.

As one component of the research, we have continued to develop novel multiscale methods for analyzing packet-switched data networks with bursty traffic exhibiting fractal behavior. These methods have also led to the development of complementary new multiscale traffic management strategies for more efficiently routing and serving packets in such networks. These new results are described in detail in the Ph.D. thesis of Warren Lam completed during this reporting period.

In other aspects of the ongoing research, we developed additional results on the use of nonlinear dynamics, chaos, and solitons in the design of innovative analog error-protection codes for communications applications, and developed new very low complexity adaptive coding techniques for exploiting the availability of feedback in unknown, time-varying wireless communication networks.

As a final component of the research, we have continued to develop promising new and bandwidth-efficient classes of time, frequency, and space diversity strategies for single- and multi-user wireless communication in multipath fading environments. Of particular note has been a class of new techniques we have developed, referred to as linear antenna precoding, for exploiting transmitter antenna arrays to combat the effects of fading. Other aspects of this research has focussed on the development of near-far resistant receiver structures for use with both conventional and recently-introduced spread-signature CDMA systems. In this area, we have developed novel recursive linear equalization algorithms and techniques for equalization based on iterative interference cancellation.

The results for this reporting period are described in detail in the following publications.

1. Lam, Warren M., "Multiscale Methods for the Analysis and Application of Fractal Point Processes and Queues," Ph.D. thesis, Dept. Elec. Eng. Comp. Sci., MIT, Feb. 1997.
2. B. Chen and G. W. Wornell, "Analog Error-Correcting Codes Based on Chaotic Dynamical Systems," submitted to *IEEE Trans. Commun.*, Dec. 1996.

3. A. C. Singer, A. V. Oppenheim, and G. W. Wornell, "Detection and Estimation of Multiplexed Soliton Signals," submitted Apr. 1997 to *IEEE Trans. Signal Processing*.
4. J. M. Ooi and G. W. Wornell, "Fast Iterative Coding Techniques for Feedback Channels: Finite State Channels and Universal Communication" submitted to *IEEE Trans. Inform. Theory*, May 1997.
5. S. H. Isabelle and G. W. Wornell, "Efficient Multiuser Detectors for Unknown Intersymbol Interference Channels," in *Proc. Signal Processing Workshop on Signal Processing Advances in Wireless Commun.*, (Paris), Apr. 1997.
6. S. Beheshti and G. W. Wornell, "Iterative Interference Cancellation and Decoding for Spread-Signature CDMA Systems," in *Proc. Vehic. Tech. Conf.*, (Phoenix), May 1997.
7. J. M. Ooi and G. W. Wornell, "Fast Iterative Coding for Feedback Channels," to appear in *Proc. IEEE Int. Sympo. Inform. Theory*, (Ulm, Germany), June 1997.
8. G. W. Wornell and M. D. Trott, "Efficient Signal Processing Techniques for Exploiting Transmit Antenna Diversity on Fading Channels," to appear in *IEEE Trans. Signal Processing*, Special Issue on Signal Processing Advances in Communications, Jan. 1997.
9. A. Narula, M. D. Trott, and G. W. Wornell, "Information-Theoretic Analysis of Multiple-Antenna Transmission Diversity for Fading Channels," in *Proc. Int. Symp. Inform. Theory and Appl.* (Victoria, Canada), Sept. 1996.
10. B. Chen and G. W. Wornell, "Efficient Channel Coding for Analog Sources using Chaotic Systems" in *Proc. IEEE GLOBECOM*, (London), Nov. 1996.
11. J. M. Ooi and G. W. Wornell, "Decentralized Control of a Multiple Access Broadcast Channel: Performance Bounds," in *Proc. Int. Conf. Dec. Control*, (Japan), Dec. 1996.
12. J. M. Ooi and G. W. Wornell, "Fast Iterative Coding Techniques for Feedback Channels," submitted Oct. 1996 to *IEEE Trans. Inform. Theory*.

13. A. Narula, M. D. Trott, and G. W. Wornell, "Information-Theoretic Analysis of Multiple-Antenna Transmission Diversity," submitted Nov. 1996 to *IEEE Trans. Inform. Theory*, Nov. 1996.
14. S. H. Isabelle and G. W. Wornell, "Recursive Multiuser Equalization for CDMA Systems in Fading Environments," in *Proc. Allerton Conf. Commun, Contr., Signal Processing*, (Illinois), Oct. 1996.

# Multiscale Representation and Estimation of Fractal Point Processes

Warren M. Lam and Gregory W. Wornell, *Member, IEEE*

**Abstract**—Fractal point processes have a potentially important role to play in the modeling of a wide range of natural and man-made phenomena. However, the lack of a suitable framework for their representation has frequently made their application in many problems difficult. We introduce natural multiscale representations for an important class of these processes based on mixtures of Poisson processes. In turn, this framework leads to efficient new algorithms for both the synthesis and the analysis of such processes. These include algorithms for optimal fractal dimension and interarrival time estimation that are of interest in a range of applications. Several aspects of the performance of these algorithms are also addressed.

## I. INTRODUCTION

**A**N extraordinary range of natural and man-made phenomena lack any characteristic temporal or spatial scale. Such phenomena and their inherent statistical scale invariance are naturally modeled using the mathematics of self-similar and fractal geometry. One set of fractal random processes that are useful in signal modeling applications are those for which the associated waveforms are continuous-valued. Important classes of these processes are  $1/f$  processes and fractional Brownian motions, and multiscale representations for these processes have proven to be extremely useful both conceptually and practically; see, e.g., [1].

Another set of fractal random processes that are useful in modeling applications are discrete-valued. Of particular interest are fractal point processes—collections of points or “events” randomly distributed temporally and/or spatially without a characteristic scale. Examples of phenomena well-modeled in this way are abundant, including the distributions of stars and planets in the universe, transmission errors on communications channels, and impulsive spikes in auditory neural signals [2], [3], [4], [5]. As we will develop in this paper, multiscale representations turn out to be an equally useful and efficient tool in the synthesis, analysis, and processing of these fractal random processes.

While no universal framework for modeling fractal point processes exists, a variety of approaches for such modeling have been pursued and developed in the literature. For example, Johnson *et al.* generate a point process with fractal

characteristics from a doubly stochastic process—specifically, a nonhomogeneous Poisson process whose arrival rate function is a suitably chosen random process with scale-invariant characteristics [6]. This framework has proven useful in modeling several aspects of the spike trains observed in auditory neural signals [7]. However, the development of efficient signal processing algorithms using this framework has traditionally proved difficult.

In a somewhat different approach, Mandelbrot constructs a fractal point process from a renewal process with interarrival times governed by a Pareto (i.e., power law) distribution [4]. To accommodate the fact that Pareto random variables are not asymptotically integrable, the concept of conditional stationarity is introduced. This model, whose second-order statistics were subsequently explored in [8], has been used to model error clustering in the telephone network [9] among other applications. However, this model is generally more useful as a conceptual characterization for some fractal point processes than as a framework for the development of signal processing algorithms. As a consequence, the fractal point process definition we develop at the outset of this paper is closely related in spirit to this model.

As the main contribution of this work, we introduce and develop a multiscale framework for modeling fractal point processes. In particular, we consider the decomposition of a fractal point process into a mixture of constituent homogeneous Poisson processes, each of which contributes the features associated with a specific scale. As we shall see, because of the inherent scale invariance of the process, there is an important statistical scaling relationship among the constituents. As we will demonstrate, this framework is especially well-suited to the development of a variety of signal processing algorithms for such fractal processes.

The outline of this paper is as follows. In Section II, we define the particular class of fractal point processes of interest in this work, and develop some of their key characteristics. In Section III, we develop multiscale constructions for these processes and some of their important features in the context of signal synthesis. In Section IV, we demonstrate how these multiscale representations are useful in addressing some fundamental signal analysis and estimation problems involving fractal point processes. Finally, Section V contains some concluding remarks.

## II. CONDITIONALLY-RENEWING FRACTAL POINT PROCESSES

We begin by developing a useful and sufficiently formal definition of the class of fractal point processes of interest

Manuscript received July 21, 1995; revised April 10, 1995. This work was supported by the Advanced Research Projects Agency monitored by ONR under Contract No. N00014-93-1-0686, and the Air Force Office of Scientific Research under Grant No. AFOSR-91-0034. The associate editor coordinating the review of this paper and approving it for publication was Prof. Banu Onural.

The authors are with the Department of Electrical Engineering and Computer Science, and the Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, MA 02139 USA.

IEEE Log Number 9415084.

# Analog Error-Correcting Codes based on Chaotic Dynamical Systems

*Brian Chen and Gregory W. Wornell*

December 15, 1996

## Abstract

The properties of chaotic dynamical systems make them useful for channel coding for a variety of practical communication applications. To illustrate this, a novel analog code based on tent map dynamics and having a fast decoding algorithm is developed for use on unknown, multiple, and time-varying SNR channels. This code is shown to be an attractive alternative to both digital codes and linear modulation in such scenarios. Several properties and interpretations of the codes are developed, along with some methods for their optimization.

*Index Terms*—chaotic systems, nonlinear dynamics, joint source and channel coding, error-correction codes, twisted modulation, broadcast channels, fading channels

## 1 Introduction

In many communication applications, the information to be transmitted over the channel of interest is inherently analog (i.e., continuous-valued) in nature. Among many examples are speech, audio, or video information. For unreliable channels, the goal is typically to encode the information at the transmitter so as to allow reconstruction at the receiver with the minimum possible distortion. Over the last few decades, there has been an increasing bias towards digital solutions to this problem. A traditional digital approach involves appropriately quantizing the source data and encoding the quantized data using a suitably designed channel code so that the quantized data can be recovered with arbitrarily low probability of error.

---

This work has been supported in part by the Defense Advanced Research Projects Agency monitored by ONR under Contract No. N00014-93-1-0686, the Air Force Office of Scientific Research under Grant No. F49620-96-1-0072, the Office of Naval Research under Grant No. N00014-96-1-0930, and a National Defense Science and Engineering Graduate Fellowship.

The authors are with the Department of Electrical Engineering and Computer Science and the Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, MA 02139.

# Detection and Estimation of Multiplexed Soliton Signals

*Andrew C. Singer, Alan V. Oppenheim, and Gregory W. Wornell*

April 16, 1997

## Abstract

Solitons are eigenfunction solutions to certain nonlinear wave equations that arise in a variety of natural and man-made systems. Their rich dynamics and tractable mathematical structure make them an intriguing component of such systems, often describing large scale or long term behavior of natural systems, or the information content in certain communication or signal processing systems. However, it is often difficult to detect or estimate the parameters of solitons in such systems due to the presence of strong non-soliton components, or due to the nonlinear interaction of multiple solitons. The objective of this paper is to develop and investigate the detection and estimation of soliton signals. As a framework for this study, we consider using these nonlinear systems as both signal generators and signal processors in a form of multiplexed soliton communication. In contrast to more conventional uses of solitons in a communications context, our communication system uses soliton systems for signal generation and multiplexing for transmission over traditional linear channels. In addition to their mathematical tractability and the simplicity of the analog circuits used to generate and process them, we show that the soliton signal dynamics may also provide a mechanism for decreasing transmitted signal energy while enhancing signal detection and parameter estimation performance.

## 1 Introduction

Solitons are stable, mode-like solutions to a class of nonlinear wave equations. The derivation of a theoretical solution to one such equation in [1] launched the development of the “inverse scattering transform,” which enables the analytic solution of a diverse class of nonlinear wave equations [2]. The inverse scattering transform can be interpreted as a nonlinear Fourier analysis for these systems which decomposes wave dynamics into a superposition of normal modes which interact nonlinearly. These normal modes are solitons, and their particle-like properties have been observed in a variety of natural phenomena including ocean and plasma waves [3][4], crystal lattice vibrations [5] and energy transport in proteins [3]. Solitons also describe the behavior of a variety of man-made systems, including super-conducting transmission lines [6], nonlinear circuits [7][8], ultrafast electronics and optoelectronics [9] and surface acoustic wave devices [10]. Solitons are currently of particular interest to the electrical engineering community because of their potential



# Fast Iterative Coding Techniques for Feedback Channels: Finite-State Channels and Universal Communication

*James M. Ooi and Gregory W. Wornell*

May 15, 1997

## Abstract

A class of practical, low-complexity, variable-rate coding schemes is developed for communication over known and unknown discrete finite-state channels with noiseless feedback. These schemes, which are based on the compressed-error cancellation framework described in [1], require a bounded number of computations per channel input for both encoding and decoding and have error probabilities that decay exponentially with average decoding delay at any rate below the mutual information induced by input distributions in the class of finite-order Markov distributions. For unknown finite-state channels, the scheme we develop can achieve the mutual information induced by the realized channel without knowledge of the channel at either the transmitter or the receiver.

*Keywords*—error-correction coding, feedback channels, iterative coding, channels with memory, finite-state channels, universal source coding, universal decoding.

## 1 Introduction

The availability of feedback in a communication system—i.e., a channel from receiver to transmitter through which the receiver passes the transmitter its observations—generally enables schemes for communicating over the forward channel to have lower computational complexity<sup>1</sup>, higher capacity, higher reliability, or a combination of these advantages, in

---

This work has been supported in part by the Advanced Research Projects Agency monitored by ONR under Contract No. N00014-93-1-0686, the Office of Naval Research under Grant No. N00014-96-1-0930, and an AT&T Doctoral Fellowship.

The authors are with the Department of Electrical Engineering and Computer Science, and the Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, MA 02139.

<sup>1</sup>Throughout this paper, we use the usual measure of computational complexity, i.e., the order of growth of the number of basic operations, including arithmetic and memory retrievals. This model of computation is appropriate for random-access machines under the uniform cost assumption [2].

# Fast Iterative Coding Techniques for Feedback Channels

*James M. Ooi and Gregory W. Wornell*

October 16, 1996

## Abstract

A class of practical, low-complexity, variable-rate coding schemes is developed for communication over channels with feedback. It is shown that for arbitrary discrete memoryless channels with noise-free feedback, these schemes achieve error probabilities that decay exponentially with blocklength at any rate below the channel capacity. Moreover, the error exponent associated with these schemes is shown to be higher than the random-coding exponent. Extensions of the strategy for use on channels with memory, unknown channels (universal decoding), and channels with noisy and delayed feedback are also developed.

*Keywords*—error-correction coding, feedback channels, iterative coding, source coding.

## 1 Introduction

Many communication links are inherently bidirectional, supporting the two-way exchange of voice, video, and other data between a pair of users. In such scenarios, a natural feedback path exists for each user's transmission, and as is well known this feedback path can generally be exploited to improve overall system performance in a variety of ways. Indeed, almost all duplex communication links in widespread use today exploit the availability of feedback, usually via an automatic repeat-request (ARQ) or related protocol.

In a typical feedback communication system, the transmitter sends data to the receiver over a noisy *forward channel*, and receives information about what the receiver actually observes via a *feedback channel*. The feedback path is often a relatively noise-free link,

---

This work has been supported in part by the Advanced Research Projects Agency monitored by ONR under Contract No. N00014-93-1-0686, the Office of Naval Research under Grant No. N00014-96-1-0930, and an AT&T Doctoral Fellowship.

The authors are with the Department of Electrical Engineering and Computer Science, and the Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, MA 02139.

# ITERATIVE INTERFERENCE CANCELLATION AND DECODING FOR SPREAD-SIGNATURE CDMA SYSTEMS

*Soosan Beheshti and Gregory W. Wornell*

Research Laboratory of Electronics  
Massachusetts Institute of Technology  
Cambridge, MA 02139 USA

**Abstract** — Spread-signature code-division multiple-access systems are designed to allow both time and frequency diversity to be exploited in multiuser wireless systems operating in fading environments. Computationally efficient multipass demodulation and decoding algorithms are developed for use in receivers with these systems. These algorithms efficiently suppress both intersymbol and interuser (multiple-access) interference to achieve a substantial diversity benefit and good near-far resistance characteristics. Moreover, it is shown that relatively few iterations are required for convergence to typical target bit-error rates. Several other aspects of the performance of the algorithms are also explored.

## I. INTRODUCTION

Spread-signature code-division multiple-access (CDMA) systems were recently introduced as an attractive alternative to conventional CDMA systems for use in time-varying multipath environments [1] [2]. Using long signatures in an overlapped manner for successive symbols, spread-signature CDMA can achieve a substantial temporal diversity benefit. Furthermore, the broadband nature of the signatures allows an additional spectral diversity benefit to be simultaneously realized.

In [1], computationally very efficient linear receivers were developed for use in conjunction with spread-signature CDMA. With such receivers, it was shown that spread-signature CDMA effectively transforms the multiuser Rayleigh fading channel into a decoupled set of additive white quasi-Gaussian noise channels. In particular, both the intersymbol and interuser (i.e., multiple-access) interferences are transformed into a second quasi-Gaussian noise source that is effectively white and uncorrelated with the input data stream.

Improved performance can be obtained by more carefully exploiting the digital (finite alphabet) nature of the users' data streams, which makes the intersymbol and interuser interferences highly structured. In principle, direct maximum likelihood (ML) decoding algorithms can be employed for optimum mitigation of such interference at the receiver. However, the associated Viterbi algorithms are invariably impractical, even for conventional CDMA systems. Indeed, ML interuser interference cancellation has complexity that grows exponentially with the number of users [3]. Moreover, ML intersymbol interference cancellation is often computationally unwieldy even for single-user systems [4].

In this paper, we describe efficient techniques for suppressing such interference at the receiver during demodulation and decoding. These algorithms explicitly take into account the structure in the interference and result in a near-far resistant receiver, but avoid the cumbersome complexity of ML decoding. In single-user scenarios, spread-signature CDMA specializes to the class of spread-response precoding algorithms described in [2]. In this case the interference suppression algorithms developed in this paper specialize to an efficient variant of the novel multistage receivers described by Wittneben in [5], for which useful new insights are obtained.

## II. SYSTEM MODEL

We consider a single cell of a typical cellular multiple-access channel in which there is a single base station and  $M$  mobiles. Both forward link (base-to-mobile) and reverse link (mobile-to-base) communication are of interest, and the users share a total fixed bandwidth of  $M\mathcal{W}_0$ , where  $\mathcal{W}_0$  is the bandwidth per user. The information to be sent by the  $m$ th user is a white  $N$ -PSK stream of (possibly coded) symbols  $x_m[n]$ , each with energy  $\mathcal{E}_m$ . The discrete-time transmission  $y_m[n]$  for the  $m$ th user is obtained by upsampling  $x_m[n]$  by a factor of  $M$ , followed by linear time-invariant filtering with the associated signature sequence  $h_m[n]$ , i.e.,

$$y_m[n] = \sum_k x_m[k] h_m[n - kM], \quad (1)$$

The signatures  $h_m[n]$  form an orthonormal set, i.e.,

$$\sum_k h_i[k - nM] h_l[k - mM] = \delta[n - m] \delta[i - l], \quad (2)$$

so that all the symbols of all the users are modulated on orthogonal, unit-energy waveforms. In spread-signature systems, these signatures have the further property that their length  $K$  is much larger than the intersymbol period  $M$ . Choosing larger spreading factors  $K$  allows larger temporal diversity benefits to be achieved. While much of our development applies more broadly, in experiments we use the so-called maximally-spread signatures [1], which are binary valued ( $h_m[n] = \pm K^{-1/2}$ ) and particularly attractive for practical implementations.

The multiuser channel is a time- and frequency-selective Rayleigh fading with stationary, uncorrelated scattering. In the equivalent discrete-time baseband model, the signal obtained at a particular receiver takes the form

$$r[n] = \sum_m \sum_k a_m[n; k] y_m[n - k] + w[n], \quad (3)$$

This work has been supported in part by DARPA monitored by ONR under Contract No. N00014-93-1-0686, the Office of Naval Research under Grant No. N00014-96-1-0930, the National Science Foundation under Grant No. MIP-9502885, and Army Research Laboratory under Cooperative Agreement DAAL01-96-2-0002.

# Multiscale Representation and Estimation of Fractal Point Processes

Warren M. Lam and Gregory W. Wornell, *Member, IEEE*

**Abstract**—Fractal point processes have a potentially important role to play in the modeling of a wide range of natural and man-made phenomena. However, the lack of a suitable framework for their representation has frequently made their application in many problems difficult. We introduce natural multiscale representations for an important class of these processes based on mixtures of Poisson processes. In turn, this framework leads to efficient new algorithms for both the synthesis and the analysis of such processes. These include algorithms for optimal fractal dimension and interarrival time estimation that are of interest in a range of applications. Several aspects of the performance of these algorithms are also addressed.

## I. INTRODUCTION

**A**N extraordinary range of natural and man-made phenomena lack any characteristic temporal or spatial scale. Such phenomena and their inherent statistical scale invariance are naturally modeled using the mathematics of self-similar and fractal geometry. One set of fractal random processes that are useful in signal modeling applications are those for which the associated waveforms are continuous-valued. Important classes of these processes are  $1/f$  processes and fractional Brownian motions, and multiscale representations for these processes have proven to be extremely useful both conceptually and practically; see, e.g., [1].

Another set of fractal random processes that are useful in modeling applications are discrete-valued. Of particular interest are fractal point processes—collections of points or “events” randomly distributed temporally and/or spatially without a characteristic scale. Examples of phenomena well-modeled in this way are abundant, including the distributions of stars and planets in the universe, transmission errors on communications channels, and impulsive spikes in auditory neural signals [2], [3], [4], [5]. As we will develop in this paper, multiscale representations turn out to be an equally useful and efficient tool in the synthesis, analysis, and processing of these fractal random processes.

While no universal framework for modeling fractal point processes exists, a variety of approaches for such modeling have been pursued and developed in the literature. For example, Johnson *et al.* generate a point process with fractal

characteristics from a doubly stochastic process—specifically, a nonhomogeneous Poisson process whose arrival rate function is a suitably chosen random process with scale-invariant characteristics [6]. This framework has proven useful in modeling several aspects of the spike trains observed in auditory neural signals [7]. However, the development of efficient signal processing algorithms using this framework has traditionally proved difficult.

In a somewhat different approach, Mandelbrot constructs a fractal point process from a renewal process with interarrival times governed by a Pareto (i.e., power law) distribution [4]. To accommodate the fact that Pareto random variables are not asymptotically integrable, the concept of conditional stationarity is introduced. This model, whose second-order statistics were subsequently explored in [8], has been used to model error clustering in the telephone network [9] among other applications. However, this model is generally more useful as a conceptual characterization for some fractal point processes than as a framework for the development of signal processing algorithms. As a consequence, the fractal point process definition we develop at the outset of this paper is closely related in spirit to this model.

As the main contribution of this work, we introduce and develop a multiscale framework for modeling fractal point processes. In particular, we consider the decomposition of a fractal point process into a mixture of constituent homogeneous Poisson processes, each of which contributes the features associated with a specific scale. As we shall see, because of the inherent scale invariance of the process, there is an important statistical scaling relationship among the constituents. As we will demonstrate, this framework is especially well-suited to the development of a variety of signal processing algorithms for such fractal processes.

The outline of this paper is as follows. In Section II, we define the particular class of fractal point processes of interest in this work, and develop some of their key characteristics. In Section III, we develop multiscale constructions for these processes and some of their important features in the context of signal synthesis. In Section IV, we demonstrate how these multiscale representations are useful in addressing some fundamental signal analysis and estimation problems involving fractal point processes. Finally, Section V contains some concluding remarks.

## II. CONDITIONALLY-RENEWING FRACTAL POINT PROCESSES

We begin by developing a useful and sufficiently formal definition of the class of fractal point processes of interest

Manuscript received July 21, 1995; revised April 10, 1995. This work was supported by the Advanced Research Projects Agency monitored by ONR under Contract No. N00014-93-1-0686, and the Air Force Office of Scientific Research under Grant No. AFOSR-91-0034. The associate editor coordinating the review of this paper and approving it for publication was Prof. Banu Onural.

The authors are with the Department of Electrical Engineering and Computer Science, and the Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, MA 02139 USA.

IEEE Log Number 9415084.

# EFFICIENT MULTIUSER DETECTORS FOR INTERSYMBOL INTERFERENCE CHANNELS

Steven H. Isabelle

Gregory W. Wornell

Research Laboratory of Electronics  
Massachusetts Institute of Technology  
Cambridge, MA 02139

## ABSTRACT

A general framework is developed for addressing problems of multiuser detection in wireless communication systems operating in the presence of time- and/or frequency-selective fading due to multipath propagation. In particular, a state-space description of the canonical multiple-access system as a multiple-input multiple-output linear system is used to obtain minimum mean-square error multiuser equalizers having efficient recursive implementations in the form of Kalman filters. These algorithms effectively suppress both intersymbol and interuser (multiple-access) interference, yielding near-far resistant receivers that can reduce power control requirements. More generally, the equalizers make efficient use of all available temporal, spectral, and spatial diversity in the system, and can be used in conjunction with both conventional CDMA and more recently proposed spread-signature CDMA systems. Decision-feedback implementations of the equalizers are also described.

## 1. INTRODUCTION

In a wide-range of wireless communication applications, there is a need for users to be able to communicate efficiently and asynchronously among themselves in the presence of fading due to multipath propagation. For example, in a cellular mobile radio environment, the transmissions of the individual mobiles pass through generally distinct channels, and a noisy version of their superposition is obtained at the base station.

In such scenarios, obtaining reliable estimates of the symbols transmitted by a particular user (or all users) requires the mitigation of several sources of interference, including multiple-access interference and intersymbol interference.

From a broader perspective, reliable communication in the presence of fading requires that diversity be exploited to improve both average and worst-case performance (outage probabilities). Such diversity takes three forms: spectral, temporal, and spatial. For example, conventional CDMA systems, where all users spread their transmission over a common bandwidth, is a format designed to enable spectral diversity to be exploited. In particular, provided the total bandwidth is large compared to the coherence bandwidth of the channel, then spectral diversity can be exploited by a suitably designed receiver. In this sense, the available diversity manifests itself in the form of intersymbol interference, and the equalizer is the mechanism by which such diversity is actually exploited.

In a similar manner, spread-signature CDMA systems

[1] and spread-response precoding systems [2], in which all users spread the transmission of their symbols in time, are designed to enable temporal diversity to be exploited. In this case, too, a suitably designed equalizer allows this diversity to be exploited. Finally, the use of antenna arrays—either at the receiver [3] or transmitter [4]—in conjunction with suitably designed equalizers allows spatial diversity to be exploited.

There is a substantial literature on the problem of multiple-access interference cancellation in such systems, and a wide range of efficient algorithms have been proposed for use in receivers—see, e.g., [5] [6] [7] [8] [9]. However, in such problems it is generally assumed that the fading in the channel is flat over the total bandwidth and over the symbol duration. By contrast, there have been comparatively few results presented on multiple-access interference cancellation for use on channels without such flat fading. In this work we develop efficient linear minimum mean-square error (MMSE) equalizer structures for such scenarios, which jointly suppress both multiple-access and intersymbol interference and therefore exploit the inherent diversity. In the case, of flat fading, the resulting algorithms yield attractive recursive implementations of some familiar multiuser detection algorithms.

## 2. SYSTEM MODEL AND PROBLEM FORMULATION

Consider a passband CDMA system in which there are  $M$  users, all sharing a total fixed bandwidth  $LW_0$ , so that  $W_0L/M$  is the effective bandwidth per user. In the equivalent discrete-time baseband model for the system, the modulation process can be viewed as follows. The coded symbol stream of the  $m$ th user ( $1 \leq m \leq M$ ), which we denote by  $x_m[n]$ , is modulated onto a unique signature sequence  $h_m[n]$  to produce  $y_m[n]$  which is transmitted within the total available bandwidth.

Conceptually, it is convenient to view the modulation process in two stages. As depicted in Fig. 1, these stages correspond to upsampling (i.e., zero-insertion) by a factor  $L \geq M$ , followed by linear time-invariant filtering with the signature sequence, i.e.,

$$y_m[n] = \sum_k x_m[k] h_m[n - kL]. \quad (1)$$

We emphasize at the outset that we avoid imposing any constraint on the length  $K$  of the signature sequences in this system. In so doing, our results will apply not only to conventional CDMA systems, for which  $K = L$ , but to recently proposed spread-signature CDMA systems for which  $K \gg L$  to allow the temporal diversity benefit to be realized in the presence of time-selective fading.

Often, but certainly not always, the signatures are chosen to satisfy some convenient orthogonal properties. For

This work has been supported in part by ARPA monitored by ONR under Contract No. N00014-93-1-0686, NSF under Grant No. MIP-9502885, and the ONR under Grant No. N0014-96-1-0930.

# Efficient Channel Coding for Analog Sources using Chaotic Systems

Brian Chen

Gregory W. Wornell

Department of Electrical Engineering and Computer Science, and  
Research Laboratory of Electronics  
Massachusetts Institute of Technology  
Cambridge, MA 02139

## ABSTRACT

We explore the application of chaotic sequences for encoding and transmission of analog sources over channels with unknown or multiple signal-to-noise ratios, as occur in broadcast and fading scenarios. Lower bounds on the mean-square distortion are derived for codes based on so-called tent-map dynamics, and are compared with those of other codes. For additive white Gaussian noise channels, we show there always exists a power-bandwidth regime in which this code yields lower distortion than any digital (i.e., finite-alphabet) code. We also develop and evaluate three practical decoding algorithms for efficiently exploiting these new codes on intersymbol interference channels.

## 1. INTRODUCTION

Among many other interesting properties, chaotic systems possess the sensitivity to initial condition property, i.e., state trajectories corresponding to nearby initial states diverge exponentially fast. Although prediction of the future states of a chaotic system is thus fundamentally difficult, this sensitivity is actually advantageous in the estimation of past states [1]. Hence, if one embeds information in the initial state of a chaotic system, the resulting state sequence forms a natural error correction code, where code sequences corresponding to nearby initial states eventually separate. It is this concept that we explore in this paper.

In particular, we explore the potential application of chaotic sequences for joint source-channel coding of analog, discrete-time data. A traditional digital approach to this communication problem has been to quantize the source data and encode the quantized data using some suitable channel code so that the quantized data can be recovered with arbitrarily low probability of error. Although such approaches can be optimal over additive white Gaussian noise (AWGN) channels with known signal-to-noise ratios (SNRs), this separation of lossy source coding (quantization) and channel coding is suboptimal when the SNR is unknown or when there are multiple SNRs. Indeed, the performance of these digital approaches depend crucially on being able to choose the proper number of quantization levels, which in turn depends on the SNR. See, e.g., Trott [2] for a discussion of several aspects of the suboptimality of separate source and channel coding for the broadcast channel.

This work has been supported in part by ARPA monitored by ONR under Contract No. N00014-93-1-0686, the Office of Naval Research under Grant No. N00014-96-1-0930, and a National Defense Science and Engineering Graduate Fellowship.

There are a wide range of scenarios in which the need to transmit data over a channel with unknown or multiple SNRs is encountered. For example, in a broadcast context, SNR typically varies from receiver to receiver. Another application involves low-delay communication over a time-selective fading channel, where the SNR fluctuates over time in a manner that is unknown *a priori*. Motivated by the knowledge that digital channel coding of quantized data can be suboptimal in these situations, in this paper we explore the use of chaotic systems to implement analog codes for the transmission of analog source data over channels with unknown or, equivalently, multiple SNRs.

We consider first the coding of a uniformly distributed source for the AWGN channel. For simplicity of exposition, we restrict our attention to real-valued baseband channels; extensions to more typical complex equivalent baseband channels are straightforward. Fig. 1 illustrates the problem considered. The source letter  $x_0$  has a uniform density on the interval  $[-1, 1]$  and is mapped into a sequence  $x[n]$  of length  $N$ , i.e., we constrain the encoder to expand the bandwidth by a factor of  $N$ . We also constrain the average signal energy per dimension to be  $P$ :

$$P = \frac{1}{N} \sum_{n=0}^{N-1} E \{x^2[n]\}. \quad (1)$$

This sequence passes through an AWGN channel where the noise  $w[n]$  has zero mean and variance  $\sigma_w^2$ . Then, the SNR is, therefore,

$$\text{SNR} = \frac{P}{\sigma_w^2}. \quad (2)$$

Finally, the decoder estimates  $x_0$  from the channel output  $y[0], \dots, y[N-1]$ . The source-channel coding problem we consider is that of finding a code with small distortion for a given SNR and bandwidth, where the distortion measure of interest is mean-square error,  $E\{(\hat{x}_0 - x_0)^2\}$ .

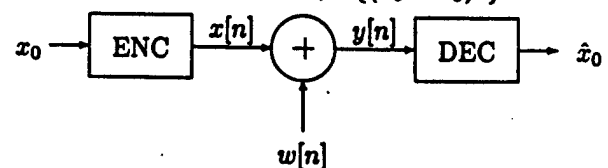


Figure 1. Joint Source-Channel Coding of a Uniform Source over an AWGN Channel

## 2. THE TENT MAP CODE

The proposed encoder maps each analog source "letter" to a chaotic sequence generated from a nonlinear dynamical

# Efficient Signal Processing Techniques for Exploiting Transmit Antenna Diversity on Fading Channels

Gregory W. Wornell, *Member, IEEE*, and Mitchell D. Trott, *Member, IEEE*

**Abstract**—A class of powerful and computationally efficient strategies for exploiting transmit antenna diversity on fading channels is developed. These strategies, which require simple linear processing at the transmitter and receiver, have attractive asymptotic characteristics. In particular, given a sufficient number of transmit antennas, these techniques effectively transform a nonselective Rayleigh fading channel into a nonfading, simple white marginally Gaussian noise channel with no intersymbol interference. These strategies, which we refer to as linear antenna precoding, can be efficiently combined with trellis coding and other popular error-correcting codes for bandwidth-constrained Gaussian channels. Linear antenna precoding requires no additional power or bandwidth and is attractive in terms of robustness and delay considerations. The resulting schemes have powerful and convenient interpretations in terms of transforming nonselective fading channels into frequency- and time-selective ones.

## I. INTRODUCTION

**S**IGNAL processing has an increasingly important role to play in wireless communications systems for a host of applications. Examples include digital cellular networks and mobile radio, wireless LAN's and wireless local loops, digital audio and television broadcasting systems, and indoor wireless and personal communication systems. Indeed, accommodating the dramatic growth in demand for such services and meeting increasingly challenging performance specifications, will require that sophisticated signal processing algorithms be an integral part of next-generation systems.

In wireless applications, fading due to multipath propagation severely impacts system performance. However, the effects of fading can be substantially mitigated through the use of diversity techniques in such systems via appropriately designed signal processing algorithms at both the transmitters and receivers. Practical, high-performance systems require that such diversity techniques be efficient in their use of resources such as power, bandwidth, and hardware cost and that they meet often stringent computational and delay constraints.

Three main forms of diversity are traditionally exploited in communication systems for fading channels: temporal, spectral, and spatial diversity.

Temporal diversity is effective when the fading is time-selective, i.e., fluctuates with time. The degree to which

this form of diversity can be exploited depends on delay constraints in the system relative to the coherence time of the fading process, which, in turn, is a function of, e.g., vehicle speeds in mobile applications. These constraints are often quite stringent for two-way voice communication but can, in principle, be significantly milder for broadcast applications. Error-correction coding [1] combined with interleaving, or precoding techniques of the type described in [2] [3], are examples of ways in which temporal diversity can be efficiently exploited.

Spectral diversity is effective when the fading is frequency-selective, i.e., varies as a function of frequency. This form of diversity can be exploited when the available bandwidth for transmission is large enough that individual multipath components can begin to be resolved. Examples of systems that take advantage of frequency diversity are direct-sequence or frequency-hopped spread-spectrum communication systems, which are designed to use wideband transmission formats.

Even in situations where the fading channel is nonselective, i.e., neither time selective nor frequency selective, or when system constraints preclude the use of these forms of temporal or spectral diversity, spatial diversity can be used to provide substantial improvement in system performance. Spatial diversity involves the use of multiple antennas sufficiently well-separated at the receiver and/or the transmitter that the individual transmission paths experience effectively independent fading. The extent to which this form of diversity can be exploited depends on issues such as cost and physical size constraints.

The use of multiple antennas at the receiver, which is referred to as receive diversity, is fairly easily exploited. In essence, multiple copies of the transmitted stream are received, which can be efficiently combined using the appropriate matched filter, i.e., maximal-ratio combining [4]. As the number of antennas increases, the outage probability is driven to zero, and the effective channel approaches an additive Gaussian noise channel, which simplifies communication. However, receive diversity can be impractical in a number of applications such as broadcasting or forward-link (base-to-mobile) transmission in cellular systems. In such scenarios, the use of multiple antennas at the transmitter, which is referred to as transmit diversity, is significantly more attractive.

Transmit diversity is, in general, less straightforward to exploit, particularly when bandwidth expansion is not feasible and when there is no feedback path to provide the transmitter

Manuscript received December 22, 1995; revised August 21, 1996. This work was supported in part by the Advanced Research Projects Agency monitored by ONR under Contract N00014-93-1-0686, the National Science Foundation under Grants MIP-9502885 and NCR-9457509, and the Office of Naval Research under Grant N00014-96-1-0930.

The authors are with the Department of Electrical Engineering and Computer Science, Massachusetts Institute of Technology, Cambridge, MA 02139 USA.

Publisher Item Identifier S 1053-587X(97)00530-8.

# Information-theoretic analysis of multiple-antenna transmission diversity for fading channels

Aradhana Narula<sup>1</sup>, Mitchell D. Trott, Gregory W. Wornell

Laboratory for Information and Decision Systems, Department of Electrical Engineering and Computer Science  
MIT 35-316, 77 Massachusetts Ave., Cambridge, MA 02139, USA.

Email: arad@image.mit.edu

**Abstract** — Several simple schemes suggested in the literature for transmitter antenna diversity are put into a common framework and compared to a theoretically optimal system. The comparison is based on mutual information as a function of antenna element gains. We show that the suboptimal schemes are uniformly worse than the optimal scheme, though the gap is small when the spectral efficiency in bits per symbol is low.

## I. INTRODUCTION

It is well-known [1] that multiple antennas can improve the performance of a communication system in a fading environment. These multiple antennas may be employed either at the transmitter or at the receiver. In a mobile radio system, it is most cost effective to employ multiple antennas at the base station and single or double antennas on the mobile units. Thus, in transmitting from the mobile to the base station, diversity is achieved through multiple receive antennas and in transmitting from the base station to the mobiles, diversity is achieved through multiple transmit antennas. In this paper we focus on transmitter diversity. A signal-processing approach to the same problem is considered in [2].

Transmitter diversity is generally viewed as more difficult to exploit than receiver diversity, in part because the transmitter is assumed to know less about the channel than the receiver, and in part because the transmitter is permitted to generate a different signal at each antenna. Unlike the receiver diversity case, where independently faded copies of a single transmitted signal may be combined optimally to achieve a performance gain, for transmitter diversity the many transmitted signals are already combined when they reach the receiver. How, then, should the transmitted signals be selected to either achieve capacity, or, more practically, to simplify the receiver while maintaining performance near capacity?

## II. CHANNEL MODEL

We model the  $M$ -antenna transmitter diversity channel as shown in Figure 1. The complex baseband received signal  $y_k = \sum_{i=1}^M \alpha_i x_{i,k} + v_k$  at time  $k$

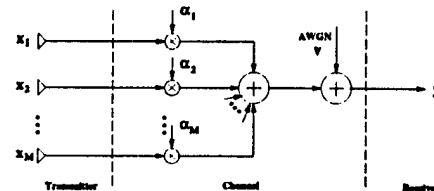


Fig. 1: Fading channel with  $M$  transmit antennas.

is the superposition of the  $M$  transmitted symbols  $x_{1,k}, \dots, x_{M,k}$ , each scaled and phase-shifted by a complex fading coefficient  $\alpha_i$  which represents the aggregate effect of the channel encountered by antenna  $i$ . The channel is frequency nonselective, i.e., the delay spread of the channel is small compared to the symbol duration. The additive noise  $v_k$  is assumed to be white circular Gaussian with variance (for each real and imaginary component)  $N_0/2$ , and the average transmitted energy is limited to  $\sum_i E|x_{i,k}|^2 \leq \mathcal{E}_s$  per symbol. As  $M$  increases the power must be distributed among the antenna elements; this allows a fair comparison of single and multiple antenna systems.

The channel is assumed to be slowly varying, so that the fading coefficients  $\{\alpha_i\}$  are effectively constant over the signaling interval of interest. The transmitter is assumed to have no knowledge of the fading coefficients, while the receiver is assumed to have perfect knowledge. We expect imprecise channel measurement to cause a smooth degradation in performance that is largely separable from other effects, though we have not established this result rigorously.

Following [3], our measure of performance is the mutual information between input and output over a long block, which corresponds in an approximate sense to the maximum achievable rate of reliable communication.

## III. DIVERSITY METHODS

We analyze five schemes for exploiting multiple transmitter antennas: "unconstrained" signaling, time division, frequency division, the time-shift technique proposed by Winters [4], and the frequency-shift technique proposed by Hiroike [5]. We will focus on the two-antenna case; the generalization to  $M > 2$  is straightforward.

By unconstrained signaling we mean that the system is evaluated as a vector-input scalar-output Gaussian channel. The other four schemes, shown in Figure 2, use linear processing to convert the vector-input channel into a scalar-input channel.

<sup>1</sup>This work was supported in part by the Department of the Air Force under contract number F19628-95-C-0002. Opinions, interpretations, conclusions, and recommendations are those of the authors and are not necessarily endorsed by the United States Air Force.



# Decentralized Control of a Multiple Access Broadcast Channel: Performance Bounds

James M. Ooi and Gregory W. Wornell  
Research Laboratory of Electronics  
Massachusetts Institute of Technology  
Cambridge, Massachusetts 02139 USA

## Abstract

Optimal decentralized control of the multiple access broadcast channel is considered. A technique is presented for upper bounding the throughput of a slotted multiple access system with a finite number of users, immediate ternary feedback, retransmission of collisions, and no buffering. The upper bound is calculated for the two- and three-user cases, and it is shown that Hluchyj and Gallager's optimized window protocol is effectively optimal for these cases.

## 1 Introduction

The multiple access broadcast channel (MABC) is a useful model for a variety of packet switched communication systems. For this channel, there has been considerable interest in the development of efficient protocols for coordinating the data transmissions of the users. Although several variations of the MABC have been considered in the literature (see, e.g., [1]—[7]), we focus on a finite-user slotted system with immediate ternary feedback, retransmission of collisions, no buffering, and no communication among users; we refer to this system as the *canonical* system.

Previous research has exploited the fact that the MABC protocol design problem can be analyzed as a decentralized control problem [2]—[7]. However, it has generally been necessary to adopt simplifications that make the problem tractable but also removed from the canonical problem. For example, Schoute [6] and Varaiya [7] both consider decentralized control of the MABC under a delayed sharing pattern and under the assumption that colliding packets incur a fixed cost rather than requiring retransmission; Rosberg [5] also assumes a fixed collision cost and no retransmissions, but differs by assuming no information sharing among controllers, as well as control inputs that depend only on broadcast feedback. Although these two simplified problems are easier to analyze, their relationships to the canonical problem are unclear. On the other hand, the simplified problems considered by Hluchyj and Gallager [4], Grizzle et al. [2], and Paradis [3] yield solutions that can be used to find lower and upper bounds on the throughput of the canonical system. Hluchyj and Gallager consider the canonical system and find protocols that are optimal in the class of protocols known as the window protocols. Since window pro-

ocols are a subset of the set of all protocols, the throughput of Hluchyj and Gallager's optimized window protocol provides a *lower* bound to the throughput achievable in the canonical system. Conversely, Grizzle et al. and Paradis attack the problem of optimally controlling a MABC that is canonical except for a one-step delay sharing (OSDS) information pattern. Because the canonical system does not allow any communication among users, its throughput is *upper* bounded by the throughput achievable under OSDS. For two users, the OSDS bound is very close to the throughput of the Hluchyj-Gallager optimized window protocol [2]—[4]. Unfortunately, for more than two users, the OSDS bound is close to neither the throughput of the Hluchyj-Gallager protocol nor any other known protocol.

We introduce new upper bounds on the throughput of the canonical system via the  $K$ -step delay state information pattern, which is similar to the previously considered  $K$ -step delay sharing pattern [8]. That is, we calculate the throughput of a MABC that is canonical except for a  $K$ -step delay state information pattern, and this provides an upper bound on the throughput of the canonical system. We use the dynamic programming method developed by Aicardi et al. [9] as a starting point for performing this calculation. However, for the class of systems that includes the MABC with  $K$ -step delay state information, this method is unnecessarily expensive in terms of computation. For this class, we develop a more computationally efficient version of the algorithm that extends its practical utility. As a result of the more efficient algorithm, we are able to find upper bounds on the throughput of the canonical system for two and three users that are tighter than the OSDS bound. The new bounds show that the performance of Hluchyj and Gallager's optimized window protocol is optimal for two users and at least nearly optimal for three users, where optimality is with respect to maximizing throughput. In fact, the Hluchyj-Gallager protocol meets the upper bound for three users when the packet arrival probability is moderately large.

The bounding technique we present is quite flexible and can be adapted to handle a greater number of users as well as other variations on the problem. Indeed, the bounding technique applies to the general class of decentralized control problems with no information sharing and may be useful in a broad range of other decentralized control and multiple-access communication problems. For this reason, we first describe the bounding technique in this general setting and later focus attention on the MABC.

This work has been supported in part by the Advanced Research Projects Agency monitored by ONR under Contract No. N00014-93-1-0686, the Office of Naval Research under Grant No. N00014-95-1-0834, and an AT&T Doctoral Fellowship.

# Fast Iterative Coding Techniques for Feedback Channels

*James M. Ooi and Gregory W. Wornell*

October 16, 1996

## Abstract

A class of practical, low-complexity, variable-rate coding schemes is developed for communication over channels with feedback. It is shown that for arbitrary discrete memoryless channels with noise-free feedback, these schemes achieve error probabilities that decay exponentially with blocklength at any rate below the channel capacity. Moreover, the error exponent associated with these schemes is shown to be higher than the random-coding exponent. Extensions of the strategy for use on channels with memory, unknown channels (universal decoding), and channels with noisy and delayed feedback are also developed.

*Keywords*—error-correction coding, feedback channels, iterative coding, source coding.

## 1 Introduction

Many communication links are inherently bidirectional, supporting the two-way exchange of voice, video, and other data between a pair of users. In such scenarios, a natural feedback path exists for each user's transmission, and as is well known this feedback path can generally be exploited to improve overall system performance in a variety of ways. Indeed, almost all duplex communication links in widespread use today exploit the availability of feedback, usually via an automatic repeat-request (ARQ) or related protocol.

In a typical feedback communication system, the transmitter sends data to the receiver over a noisy *forward channel*, and receives information about what the receiver actually observes via a *feedback channel*. The feedback path is often a relatively noise-free link,

---

This work has been supported in part by the Advanced Research Projects Agency monitored by ONR under Contract No. N00014-93-1-0686, the Office of Naval Research under Grant No. N00014-96-1-0930, and an AT&T Doctoral Fellowship.

The authors are with the Department of Electrical Engineering and Computer Science, and the Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, MA 02139.

# ATTACHMENT NUMBER 1

## REPORTS AND REPORT DISTRIBUTION

### REPORT TYPES

- (a) Performance (Technical) Report(s) (Include letter report(s)) Frequency: Annual
- (b) Final Technical Report, issued at completion of Grant.  
**NOTE:** Technical Reports must have a SF-298 accompanying them.
- (c) Final Financial Status Report (SF 269)
- (d) Final Patent Report (DD 882)

REPORTS DISTRIBUTION		
ADDRESSEES	REPORT TYPES	NUMBER OF COPIES
Office of Naval Research Program Officer Rabinder N. Madan ONR 313 Ballston Centre Tower One 800 North Quincy Street Arlington, VA 22217-5660	(a) & (b) w/(SF-298's)	3
Administrative Grants Officer OFFICE OF NAVAL RESEARCH REGIONAL OFFICE BOSTON  495 SUMMER STREET ROOM 103 BOSTON, MA 02210-2109	(c), (d) & SF-298's only for (a) & (b)  *	1
Director, Naval Research Laboratory Attn: Code 2627 4555 Overlook Drive Washington, DC 20375-5326	(a) & (b) w/(SF-298's)	1
Defense Technical Information Center 8725 John J. Kingman Road STE 0944 Ft. Belvoir, VA 22060-6218	(a) & (b) w/(SF-298's)	2
Office of Naval Research Attn: ONR 00CC1 Ballston Centre Tower One 800 North Quincy Street Arlington, VA 22217-5660	(d)	1

If the Program Officer directs, the Grantee shall make additional distribution of technical reports in accordance with a supplemental distribution list provided by the Program Officer. The supplemental distribution list shall not exceed 250 addresses.

\* For report types (a) and (b), send only a copy of the transmittal letter to the Administrative Contracting Officer; do not send actual reports to the Administrative Contracting Officer.